

Density Perturbations from Two-field Inflation¹

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Abstract. We discuss metric perturbations produced during a period of inflation in the early universe where two scalar fields evolve. The final scalar perturbation spectrum can be calculated in terms of the perturbed expansion along neighbouring trajectories in field-space. In the usual single field case this is fixed by the values of the fields at horizon-crossing, but in the presence of more than one field there is no longer a unique slow-roll trajectory. The presence of entropy as well as adiabatic fluctuations means that the super-horizon-sized metric perturbation ζ may no longer be conserved and the evolution must be integrated along the whole of the subsequent trajectory. In general there is an inequality between the ratio of tensor to scalar perturbations and the tilt of the gravitational wave spectrum, which becomes an equality when only adiabatic perturbations are possible and ζ is conserved.

Inflation was originally proposed as a mechanism to produce a flat, isotropic and homogeneous universe. However it was soon realised that vacuum fluctuations in a scalar field driving inflation would also lead to the production of an approximately scale-invariant spectrum of inhomogeneities. A good deal of progress has been made in understanding and parameterising generic features of the perturbation spectrum independently of the specific interaction potential as long as it is driven by a single scalar field [1]. Our intention here is to emphasise which features are restricted to models of a single field evolving during inflation and what happens when more than one field is present [2, 3].

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In single field inflation there is a single attractor trajectory, the non-decaying mode for the homogeneous solution for the inflaton field, $\phi(t)$. This has two important consequences. Firstly, we can associate any particular time during inflation with a particular value of the scalar field. Secondly, the non-decaying mode of any perturbation on large scales (to be defined below) corresponds to a perturbation in time along this homogeneous solution, $\delta t \equiv \delta\phi/\dot{\phi}$.

We can always decompose the full time- and spatially-dependent inflaton field into Fourier modes with comoving wavenumbers k , which evolve independently in the linear approximation. Because the comoving Hubble length, H^{-1}/a , shrinks during inflation any Fourier mode will eventually be stretched far beyond the horizon, even though it may start far within the Hubble scale. Assuming each mode starts in the Bunch-Davies vacuum state on small scales ($k \gg aH$), they will have an amplitude $\delta\phi \simeq H/2\pi$ at horizon crossing², on spatially flat hypersurfaces. If one makes a gauge transformation to a comoving hypersurface (one with uniform ϕ) we find the intrinsic curvature perturbation is $^{(3)}\delta R \equiv 4\mathcal{R}k^2/a^2$, where the metric perturbation, \mathcal{R} , is a particularly useful quantity. It approaches a constant value on large scales [4],

$$\mathcal{R} \rightarrow \zeta = \left(\frac{H\delta\phi}{\dot{\phi}} \right) \quad \text{as} \quad \frac{k}{aH} \rightarrow 0, \quad (1.1)$$

which is simply the perturbation in the number of expansion times, $\zeta \equiv \delta N = H\delta t$, where $\delta t \equiv \delta\rho/\dot{\rho}$. It is important to emphasise that the constancy of ζ on super-horizon scales is not dependent on the slow-roll approximation, but is simply a consequence of the perturbations being adiabatic. Indeed we can write down an expression for the time derivative of ζ [3]

$$\dot{\zeta} = 3H \left(\frac{\dot{p}}{\dot{\rho}} - \frac{\delta p}{\delta\rho} \right) \zeta, \quad (1.2)$$

in the limit $k/aH \rightarrow 0$, for a general fluid with pressure p and density ρ . Adiabatic perturbations can be represented by a time perturbation $\delta t = \delta p/\dot{p} = \delta\rho/\dot{\rho}$. So long as there is a single fluid with any equation of state, $p(\rho)$, the perturbations will be adiabatic and thus ζ is conserved after inflation during the radiation or matter dominated eras until aH has decreased back to the value k and the mode re-enters the horizon [5]. The density contrast at re-entry is given by $(\delta\rho/\rho)_{k=aH} = (2/5)\zeta$ during the matter dominated era [1]. Even though the physics of re-heating the universe at the end of inflation may be exceedingly complicated, ζ is conserved on super-horizon scales when changes in the equation of state occur at a fixed energy density, since ζ gives the intrinsic curvature perturbation on the boundary [6].

Each mode k has a single horizon-crossing time during inflation when $aH = k$ and the field ϕ has a value ϕ_* . If we can determine $\zeta(k)$ from observations of the microwave background sky or large-scale structure, one can set about reconstructing the inflaton potential $V(\phi)$ when these scales left the horizon, independently not only of the details of re-heating, but also regardless of the latter stages of inflation which can only affect smaller scales.

²Strictly speaking, this is the perturbation in the non-decaying asymptotic solution, evaluated when $aH = k$ for a quasi-massless field ($m^2 \ll H^2$).

It is straightforward to generalise Eq. (1.1) to the case of two scalar fields in the slow-roll limit,

$$\zeta \simeq H \left(\frac{\dot{\phi}\delta\phi + \dot{\sigma}\delta\sigma}{\dot{\phi}^2 + \dot{\sigma}^2} \right) \quad \text{as} \quad \frac{k}{aH} \rightarrow 0, \quad (1.3)$$

where ζ now represents the metric perturbation on uniform density hypersurfaces. However this quantity is not constant on super-horizon scales due to the presence of entropy perturbations. There is no longer a locally defined time perturbation since $\delta\phi/\dot{\phi} \neq \delta\sigma/\dot{\sigma}$ in general, and we have

$$\dot{\zeta} = \frac{H}{2} \left(\frac{\delta\phi}{\dot{\phi}} - \frac{\delta\sigma}{\dot{\sigma}} \right) \frac{d}{dt} \left(\frac{\dot{\phi}^2 - \dot{\sigma}^2}{\dot{\phi}^2 + \dot{\sigma}^2} \right). \quad (1.4)$$

To know the value of ζ at the end of, or after, inflation we must integrate ζ along the whole subsequent trajectory and establish the final perturbation in the expansion time. This is not determined simply by the perturbation at horizon crossing because different regions of the universe could subsequently follow radically different trajectories. With two fields present there may be many different possible trajectories during inflation. Even if we restrict our analysis to the slow roll approximation, which reduces the four-dimensional phase-space to a two-dimensional field-space, the end of inflation surface, for instance, now becomes a line rather than a single point in field space.

On the other hand, if one can construct the function $N(\phi, \sigma) = \int_{\gamma} H(\phi, \sigma) dt$ giving the total expansion from any point to the end of inflation along each classical trajectory γ , one can still in principle give $\zeta_e \equiv \delta N$ in terms of the perturbation at horizon crossing [2]. This is of course equivalent to integrating the perturbation along the subsequent trajectory but gives an intuitively simpler picture and allows one to work with only the homogeneous field equations. In previous work we have considered the perturbation spectra generated during inflation in models with a separable interaction between two scalar fields [3], a particular case of which is scalar-tensor gravity where a dilaton (or Brans-Dicke) field is non-minimally coupled to the metric [7].

If the second scalar field is held fixed ($\dot{\sigma} = 0$) in a local minimum of the potential then equation (1.3) reduces to equation (1.1). In practice there is a single trajectory and we can easily construct $N(\phi)$. This occurs in most models of extended [8] or hybrid [9] inflation where σ has a large positive effective mass until some critical point ϕ_c , where σ undergoes a phase transition and inflation ends. However this relies on inflation ending rapidly at a first- or second-order phase transition. Recently [10] it has been argued that in a more “natural” model of hybrid inflation the two fields should have similar bare masses $|m^2| \sim H^2 \sim 1\text{TeV}$. If so, the phase transition may not complete rapidly and scales crossing outside the horizon near to the phase transition may correspond to large scales today. There is certainly more than one possible trajectory close to ϕ_c where σ is effectively massless, and perturbations need not be purely adiabatic. Fortunately it is possible to give an approximate form for N after the critical point [11] and we can also show that all the trajectories converge on a single trajectory before the end of inflation. Thus by the end of inflation we are left with adiabatic perturbations along this trajectory. This is important as otherwise the final spectrum could depend upon the detailed dynamics of re-heating, as mentioned above.

In addition to the scalar perturbations, there is always a spectrum of tensor perturbations (gravitational waves) generated during inflation [12], whose amplitude is proportional to H at horizon crossing. The tilt of this spectrum is $n_T = 2\dot{H}/H^2$ irrespective of the matter content. Detection of such a spectrum of gravitational waves with $-2 < n_T < 0$ would be a definitive test of inflation. In the case of single field inflation, n_T is related to the ratio of tensor to scalar perturbations of the microwave background on large angular scales, $R \simeq 6|n_T|$ [1]. This has been proposed as a test of inflation, but in fact this is a characteristic only of purely adiabatic perturbations generated during inflation where ζ is constant on super-horizon scales. In the presence of entropy perturbations during inflation the equality becomes an inequality $R < 6|n_T|$ [13, 2, 3].

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